

Problem Set 6

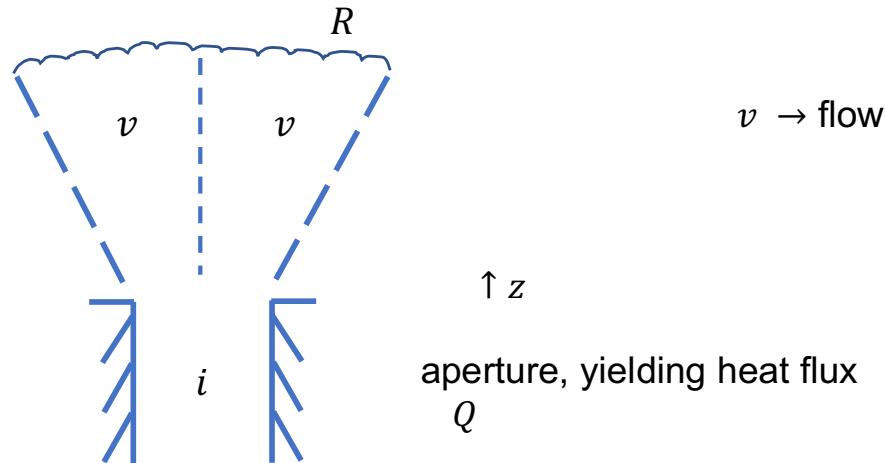
Due Thursday, March 15

- 1) Tapas: Keep your answers here brief and focused.
 - a) Estimate the number of modes needed to simulate a high Re flow, assuming you resolve the dissipation scale (many simulations don't!).
 - b) Estimate the Nusselt number for turbulent boundary layer flow. What is the ratio of Nu to that for the corresponding laminar case?
 - c) Briefly describe the layer structure of turbulent flow over a rough surface with roughness scale y_0 .
 - d) Does the continuum model of a fluid improve or decline in validity as Re increases? Hint: Compare the mean free path to the dissipation scale. Take the flow subsonic.
 - e) Calculate the spectrum of Burgers turbulence, assuming the flow may be written as a superposition of uncorrelated shocks. Hint: This is a 'three liner'. Consider a convenient way to write the flow v .

- 2) Derive the coupled equations for the spectrum density $|v_k|^2$ and response function for Burgers turbulence, which is randomly forced at large scale. The posted notes on closures and the posted articles by Kraichnan, and Forster et al. may be useful. Your answer should include:
 - a) A spectral energy transfer equation for $\partial_t |v_k|^2$, including propagators and triad coherence time.
 - b) A response function, with amplitude dependent effective (turbulent) viscosity
 - c) A triad (3 mode) coherence time, which appears in the for the $\partial_t |v_k|^2$ equation and which depends upon the turbulent viscosity.

- 3) Continuing with the system from Problem 2:
 - a) Show your result for the spectral energy equation conserves energy. Is this a discriminating test of the renormalized theory?
 - b) What physically, does the k dependency of the eddy viscosity mean?
 - c) What is the eddy viscosity at large scales? Give a simple derivation.
 - d) Are the behavior and dependency of the triad coherence time physically reasonable? How might you improve them?
 - e) If one introduces a coupling coefficient λ in the nonlinear term: $v\partial_x v \rightarrow \lambda v\partial_x v$ what does Galilean invariance imply about corrections to λ ?

4) Consider a plume of hot gas, rising as shown below:



The heat flux Q is carried by convection, and the plume spreads with height.

- Argue that: $Q = \rho c_p \tilde{T} v R^2 \sim \text{constant}$, and so determine the relation between \tilde{T} , height z and vertical velocity v .
- Now use the balance of fluid inertia and buoyancy force for stationary convection to eliminate $v(z)$ and calculate $\tilde{T}(z)$. $\tilde{T}(z)$ should scale as $\tilde{T}(z) \sim z^{-5/3}$.
- Calculate the scalings of $v(z)$.
- Calculate $Ra(z)$ and $Re(z)$, i.e.: Rayleigh and Reynolds number as functions of z . For what range of z is it valid to treat the plume as turbulent?

5) More Small Plates:

- a) Estimate the largest water droplet size that will survive suspended in a turbulent air flow of Reynolds number Re and integral scale L . Don't worry about gravity — i.e. assume the experiment is done in the space shuttle.
- b) Estimate the temperature of a body of size R in a flow v_0 with temperature T_0 . Hint: Consider viscous heating effects.
- c) What are the limiting forms of the dependence of Nusselt number on Prandtl number in a laminar boundary layer when $Re \gg 1$ and $Pr \gg 1$?

6) This problem asks you to apply your understanding of closure theory to 2D turbulence on a beta plane. You may find it useful to consult some of the posted articles or lecture notes along the way.

- a) Starting from the β -plane equation, derive an evolution equation for mode $\phi_k(t)$. Assume forcing is at intermediate scales.
- b) Derive an evolution equation for $|\phi_k(t)|^2$ using closure. Your result should depend on intensities and triad coherence time. Give expressions for the latter.
- c) Describe how the triad coherence time behaves for:
 - Weak turbulence (low intensity)
 - Strong turbulence (high intensity)
- d) At what scale do wave effects become important as energy cascades toward larger scales? Can you estimate this scale by simpler means? Welcome to the Rhines scale—an important emergent scale in GFD.
- e) Will it be easy to satisfy the selection rules when wave dispersion is important? What if the triad is composed of two Rossby waves and one $k_x = 0$ mode (zonal flow)?
- f) On the basis of your analysis, propose a hypothesis as to what scale sets the width in latitude of the zonal flow. Why?—support your argument.